QUASISTEADY AXISYMMETRIC JET ISSUING INTO A FLUIDIZED BED

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The external problem for a jet issuing into a fluidized bed is solved and estimates of the volume of the liquid phase between the jet and the bed are made.

A system of parallel gas jets in a preliminarily fluidized bed of particles provides a convenient means of improving the bed characteristics and intensifying transfer processes in the bed [1]. The intensity of phase interaction and other properties of the fluidized bed are greatly determined in this case by the development of jet flares and the interaction of adjacent jets. To ensure optimal conditions of the process, it is necessary to have estimates characterizing both the volume of the particles and gas between the jet and the bed and the radius of influence of a single jet on the bed structure.

The problem of determining a value characterizing the mass transfer between the axisymmetric jet and the bed was considered in [2] and more rigorously in [3] for a plane jet flow. The present work, resembling [3] in many respects, considers the external problem for an axisymmetric jet issuing into a fluidized bed, with the aim of determining the pressure field and the relative velocity of the gas in the bed in the presence of a single jet.

Applying a physical analysis similar to that in [3] to the vertical flow of a quasisteady axisymmetric jet in a fluidized bed, the problem can be reduced to the following calculational scheme. Region D_1 (see Fig. 1) is filled with the dense phase of the bed, and region D_2 is filled with an attenuated suspension; the boundary between these regions is formed by the surface of the jet channel AB and the free surface BC. Since the particle concentration in the flare is small [3], the pressure drop over the length of the flare is considerably less than that over the height of the dense phase of the bed, and hence in the first approximation the pressure at the boundary of channel AB should be assumed constant, equal to the pressure at the free surface BC of the bed.

Neglecting the inertia of the gas and the tangential stress resulting from molecular viscosity of the gas and also from random fluctuations of both phases, the equation of motion in region D_1 takes the form

$$-\nabla p-\alpha\vec{u}=0,\ \operatorname{div}(\varepsilon\vec{v})=0,\ \vec{u}=\vec{v}-\vec{w},\ \rho=1-\varepsilon,\ \alpha=\alpha(\varepsilon). \tag{1}$$

Equation (1) describes gas filtration in a mobile granular layer.

The boundary conditions for Eq. (1) follow from the requirement that the pressure at the boundary ABC be constant, the condition of impermeability of the external boundary $r = R$, which may be either the wall of the apparatus or the symmetry surface separating the region of influence of adjacent jets (see [4]), and also the requirement that the flow rate of the fluidizing gas through the gas-distributor grid be constant. The mathematical formulation of these conditions is analogous to that in [3]. Introducing the pressure p^0 and gas velocity u^0 unperturbed by the jet

$$p^0=-\alpha u^0(H-z)=\rho d_1 g(H-z),\ u_x^0=0,\ u_z^0=u^0 \tag{2}$$

and the excess pressure $\varphi = p - p^0$, and assuming the porosity in region D_1 to be constant in the first approximation, the problem takes the form

$$\Delta\varphi=0,\ \mathbf{u}=\mathbf{u}^0+\mathbf{u}',\ \mathbf{u}'=-\alpha^{-1}\nabla\varphi,\ \frac{\partial\varphi}{\partial z}=0\,(z=0); \tag{3}$$

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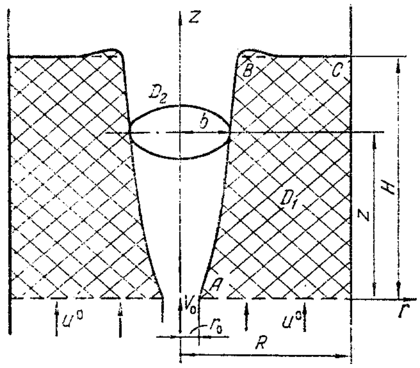

Fig. 1. Diagram for formulation of problem.

$$\frac{\partial \varphi}{\partial r}=0\,(r=R);\ \ \varphi=-\alpha u^0(H-z)\ (\mathbf{r}\in ABC),$$

and, in principle, by solving this problem it is possible to find the pressure field and relative gas velocity in the intervals between the particles when the jet is present. In solving Eq. (3) it is natural to neglect the deviation of BC from the horizontal plane z = H and the dependence on z of the coordinate b(z) describing the surface AB. It is then simple to solve Eq. (3) by separation of variables. For the axisymmetric jet

$$\varphi=-\frac{8\alpha}{\pi^2}u^0H\sum_1^n\frac{\cos\omega_n\zeta}{(2n-1)^2}\cdot\frac{K_0(\omega_n\xi)+\dfrac{K_1(\omega_n\xi_R)}{I_1(\omega_n\xi_R)}I_0(\omega_n\xi)}{K_0(\omega_n\xi_0)+\dfrac{K_1(\omega_n\xi_R)}{I_1(\omega_n\xi_R)}I_0(\omega_n\xi_0)};$$

$$u_z'=-\frac{4}{\pi}u^0\sum_1^n\frac{\sin\omega_n\zeta}{2n-1}\cdot\frac{K_0(\omega_n\xi)+\dfrac{K_1(\omega_n\xi_R)}{I_1(\omega_n\xi_R)}I_0(\omega_n\xi)}{K_0(\omega_n\xi_0)+\dfrac{K_1(\omega_n\xi_R)}{I_1(\omega_n\xi_R)}I_0(\omega_n\xi_0)};\qquad(4)$$

$$u_r'=-\frac{4}{\pi}u^0\sum_1^n\frac{\cos\omega_n\zeta}{2n-1}\cdot\frac{K_1(\omega_n\xi)-\dfrac{K_1(\omega_n\xi_R)}{I_1(\omega_n\xi_R)}I_1(\omega_n\xi)}{K_0(\omega_n\xi_0)+\dfrac{K_1(\omega_n\xi_R)}{I_1(\omega_n\xi_R)}I_0(\omega_n\xi_0)};$$

$$\omega_n=\frac{\pi}{2}(2n-1).$$

Using Eq. (4), which describes the variation in pressure and gas velocity in the bed due to the influx of the gas jet, it is possible to analyze the influence of the initial conditions of influx of a single jet or a set of jets on the structure of the bed in the vicinity of the jets. Numerical analysis of Eq. (4) shows that in engineering calculations the distributions of the velocities u_z' and u_r' may be approximated with 7% accuracy by the first 10 terms for $R \gg r_0$ and a wide range of the jet and bed parameters. The results obtained, as noted above, may also be used for the approximate analysis of a bed with a large number of parallel jets, if R is taken as half the distance between adjacent jets. Then the equation above will give an approximate description of the intensity of gas motion in the intervals between the jets.

In the case of a single jet ($R \to \infty$), Eq. (4) simplifies considerably:

$$\varphi=-\frac{8\alpha}{\pi^2}u^0H\sum_1^n\frac{\cos\omega_n\zeta}{(2n-1)^2}\cdot\frac{K_0(\omega_n\xi)}{K_0(\omega_n\xi_0)};$$

$$u_z'=-\frac{4}{\pi}u^0\sum_1^n\frac{\sin\omega_n\zeta}{2n-1}\cdot\frac{K_0(\omega_n\xi)}{K_0(\omega_n\xi_0)};\qquad(5)$$

$$u_r'=-\frac{4}{\pi}u^0\sum_1^n\frac{\cos\omega_n\zeta}{2n-1}\cdot\frac{K_1(\omega_n\xi)}{K_0(\omega_n\xi_0)}.$$

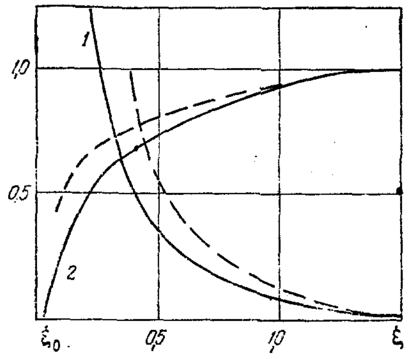

Fig. 2

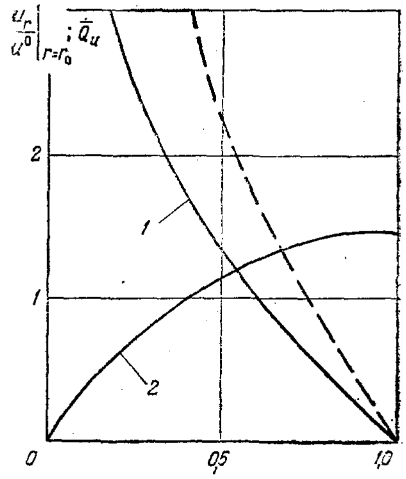

Fig. 3

Fig. 2. Dependence of dimensionless gas velocity at bed outlet in the vicinity of the jet (curve 2) and velocity of gas seepage to the base of the jet along the gas-distributor grid (curve 1) on the dimensionless horizontal coordinate $\xi = r/H$; $r_0 = 10$ mm; H = 200 mm.

Fig. 3. Distribution of radial component of relative gas velocity at the jet-channel surface and relative flux into jet: 1) $u_r/u^0|_{r=r_0}$; the dashed curve is for $r_0^! < r_0$; 2) $\bar{Q}_u = Q_u/u^0 H_\varepsilon$; $r_0 = 10$ mm; H = 200 mm; the dimensionless parameter $\zeta = z/H$ is plotted along the abscissa.

Numerical investigation shows that, if r is larger than r_0 ($r \geqslant 2r_0$), the last two series in Eq. (5) converge satisfactorily and, with ∿5% accuracy, the series may be limited to the first four terms. If r is close to r_0 ($r < 2r_0$), the series converge more slowly and the sum of the first four terms gives only a rough estimate of the velocity, which agrees with the assumption of linearity of the jet channel made in the problem. However, the reduction in accuracy in this range of r is not of practical significance, since in real situations the flare radius is considerably more than $2r_0$.

For a jet of large initial radius issuing into a low fluidized bed, when $\omega_n\xi > \omega_n\xi_0 > \omega_1\xi_0 = (\pi/2)\xi_0 \gg 1$, the asymptotic approximation of the Bessel functions in Eq. (5) is used. Summing the last two series in Eq. (5) gives

$$\frac{u_z'}{u_0} = -\frac{2}{\pi}\sqrt{\frac{\xi_0}{\xi}}\,\mathrm{arctg}\left\{2e^{-(\xi-\xi_0)}\sin\frac{\pi}{2}\zeta\right\},\quad \frac{u_r'}{u_0} = -\frac{2}{\pi}\sqrt{\frac{\xi_0}{\xi}}\,\mathrm{Arth}\left\{2e^{-(\xi-\xi_0)}\cos\frac{\pi}{2}\zeta\right\}. \tag{6}$$

Investigation of Eqs. (4)-(6) shows that there is significant ejection of gas in the jet. In Fig. 2, the dependence — according to Eq. (5) — of $u_z/u^0|_{z=H}$ (curve 2) and $u_r/u^0|_{z=0}$ (curve 1) on the dimensionless horizontal coordinate ξ is shown. The first of these parameters characterizes the relative velocity at the outlet from the fluidized bed and the second, the seepage of gas to the base of the jet channel along the gas-distributor grid. For comparison, the dashed lines in Fig. 2 show curves calculated from Eq. (6) for $r_0 = 10$ mm and H = 200 mm; even for these parameters, which do not correspond to the conditions of the approximation, the discrepancy with the accurate formula is slight. The agreement is evidently better when the channel radius is large. Accordingly, the use of Eq. (6) rather than Eq. (5) may be recommended in engineering calculations of the velocity profiles in the bed at some distance from the jet channel. It is evident from the graphs that the best accuracy (maximum error not exceeding 10%) using Eq. (6) is obtained for distances r > 0.5H.

It follows from the form of the curve of $u_r/u_0|_{z=0}$ that the jet has a considerable effect on the internal hydrodynamics of the bed, which becomes apparent at distances of the order of the bed height H, as for a plane jet [3]. However, comparison with a plane jet shows that, in contrast to the identical dependence of $u_z/u^0|_{z=H}$, the radial velocity component rises considerably more steeply as the jet axis is approached and the gas flux in the jet (injection) is more intense in the case of an axisymmetric jet.

The density of the injection gas flow in the jet may be determined as the relative gas velocity at the boundary of the jet channel $b = r_0$. From Eqs. (3) and (5)

$$u_r|_{b=r_0} = u'_r|_{b=r_0} = -\frac{4}{\pi} u^0 \sum_1^n \frac{\cos \omega_n \zeta}{2n-1} \cdot \frac{K_1(\omega_n \xi_0)}{K_0(\omega_n \xi_0)}. \quad (7)$$

This formula determines the density of the relative gas flow from the dense phase of the bed to the jet, associated with its motion relative to the disperse phase. The total bulk flow of injected gas, in the section of the jet channel from the grid to the level z, consists of the relative flux Q_u associated with the relative flow of gas and the flux Q_p corresponding to flow with the disperse-phase velocity:

$$Q_f(z) = Q_u(z) + \frac{\varepsilon}{\rho} Q_p(z). \quad (8)$$

The relative flux Q_u for an axisymmetric jet is

$$Q_u(z) = -\varepsilon 2\pi r_0 \int_0^z u_r dz = -\varepsilon 2\pi r_0 H \int_0^\zeta u_r(\zeta)\, d\zeta. \quad (9)$$

In Fig. 3 the distributions of the relative gas velocity at the boundary of the jet channel and of the relative flux of gas phase in the jet channel determined from this velocity are shown. As is evident from Fig. 3, the injected flux Q_u from the bed to the jet falls monotonically with increase in distance from the nozzle. The curve of $u_r/u^0|_{r=r_0}$ (1) is considerably steeper than in the case of a plane jet [3] and becomes steeper as the channel radius decreases. The same is true of the dependence of the relative gas flux in the jet, which, as would be expected, rises more rapidly with z in a circular jet than in a plane jet [3]. Elementary estimates of the density of the ejected flow for the plane and circular jets show that for constant relative gas flux Q, the density of the ejected flow for a plane jet of width d_0 is $Q/S \sim 2l/ld_0 \sim 2/d_0$. For a circular jet of the same diameter, $Q/S \sim 4/d_0$, i.e., twice as large as for the plane jet. This is also evident in the curves shown in Fig. 3.

An approximate estimate of the flux Q_p may be obtained in accordance with [2, 3]. A more rigorous analysis of the total bulk flux of particles in the jet Q_p, taking into account the effective viscosity of the disperse system with slip of the layers of particles in the flare, may form the subject of a separate work.

An apparatus similar to that described in detail in [7] was used for experimental verification of Eqs. (4) and (5) and of deductions from them regarding the relative radius of influence of the jet on the dense phase of the bed. The experiments were carried out under quasisteady conditions ($X_F/H = 0.8$) with two semibounded jets issuing into fluidized beds ($W \geqslant 1.1$) of aluminosilicate catalyst, polystyrene, and nitrofosk (a nitrogen—phosphorus—potassium fertilizer); narrow fractions of mean diameter 1.50-4.46 mm were used. The nozzle diameter was 6 mm; the initial jet velocity varied (depending on the bed height and the particle size) from 21.2 to 52.7 m/sec. The distance between the jets could be altered and both the development of the jets and the particle motion between the jets (in the interflare region) could be observed (visually and using moving-picture recordings).

In the case of two jets, compression of the dense phase in the interflare region as a result of partial outflow of the gas in the jet is observed. When the jets are close together, this high gas density in the vicinity of the jet leads to a considerable decrease in the local gas velocity — to values less than the initial fluidization velocity. As a result, the motion of the disperse phase close to the jet resembles the sliding of layers of free-flowing material over slip surfaces and differs sharply from particle motion in a truly fluidized system, which takes place before the jet is introduced. At the midpoint of the interflare region, a region of nonmoving particles is formed. Visual observation shows that with increase in distance between the jets there is a continuous transition from particles taking part in slipping motion in the immediate vicinity of the jet boundary (at a radius of order $0.5X_F$) to particles moving within the disperse phase of the bed. At the center of the interflare region, a region in which the influence of the jet is degenerate is formed. The minimum distance between the jets such that a region of particle motion (characteristic of a truly fluidized bed) is formed at the center of the interflare region is equal to twice the flare width. This is in good agreement with the results of the analysis of Eqs. (4) and (5) shown in Fig. 2.

In conclusion, it is appropriate to give a brief discussion of the jet parameters for which the above solution is valid. In formulating Eq. (3), the pressure at the boundary surface between the bed and the jet was assumed constant; this implies a low bulk concentration of particles in the jet channel ($\rho' \ll 1$), breakdown of the bed by the jet, and effective removal of the particles in the space above the bed.

Removal of the particles in the space above the bed means that

$$V_{z=H} > U. \tag{10}$$

If this condition is not satisfied, a quasisteady jet of the kind considered here cannot exist. In fact the particle velocity then vanishes at some cross section of the channel and "choking" of the jet begins, which leads to overlapping of the channel by particles and separation of its upper and lower parts. This is evidently the explanation for instability of jets with the regular formation of bubbles in the case of a high fluidized bed [6].

The condition $\rho' \ll 1$ means that

$$Q_p(H) \ll (V-U)\, b^2. \tag{11}$$

If this condition is not satisfied, the particle concentration in the upper part of the jet channel reaches the same order of magnitude as the concentration in the fluidized bed, i.e., local spouting begins [7]; this may be considered as the limit of reliability of Eq. (4). Note, incidentally, that if the value at the channel outlet is known, it is simple to estimate the height of particle ejection in the space above the bed, which may be important in a number of applications.

NOTATION

p, gas pressure; v, gas velocity in intervals between particles; w, velocity of disperse phase; u, relative gas velocity in intervals between particles; ρ, ρ', bulk concentrations of particles in dense phase of bed and in jet channel; ε, porosity of dense phase of bed; α, drag coefficient; φ, excess pressure; ω_n, eigenvalues; H, bed height; R, external size of bed; X_F, extent of flare (jet channel) in bed; d_0, d_1, gas and particle densities; g, acceleration due to gravity; Q_u, Q_f, Q_p, relative gas flux and total bulk fluxes of liquid and disperse phase in jet; r_0, b, initial jet radius and radius of jet channel; U, velocity of fall; V, gas velocity in jet; $\xi = r/H$, $\zeta = -z/H$, $\xi_0 = r_0/H$, $\xi_R = R/H$, dimensionless coordinates; r, z, coordinates; I_0, I_1, modified Bessel functions of zero and first orders; K_0, K_1, modified Hankel functions of zero and first orders. Indices: a superscript zero denotes gas flow in the bed unperturbed by the jet; a subscript zero, the initial jet parameters.

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